

## DOCUMENT RESUME

ED 351 291

SP 034 076

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TITLE Mathematical Literacy To Empower Teacher Education Students in the 21st Century: How Can This Become Reality?  
PUB DATE Jun 92  
NOTE 22p.; Paper presented at the National Forum of the Association of Independent Liberal Arts Colleges for Teacher Education (5th, Louisville, KY, June 5-8, 1992).  
PUB TYPE Reports - Descriptive (141) -- Speeches/Conference Papers (150)  
EDRS PRICE MF01/PC01 Plus Postage.  
DESCRIPTORS \*Constructivism (Learning); Educational Change; Elementary Secondary Education; Higher Education; \*Holistic Approach; Mathematics Education; \*Methods Courses; \*Preservice Teacher Education; \*Teacher Education Curriculum; Teacher Improvement; Teaching Methods  
IDENTIFIERS \*Maryville University of Saint Louis MO; Preservice Teachers; Reflective Teaching

## ABSTRACT

Maryville University (Missouri) believes in training competent beginning teachers who understand that all children can learn. Regarding mathematics, prospective teachers must often be convinced they can learn it themselves. Because most students do not have much math preparation, do not like it, and have great misconceptions, Maryville assumes they need to acquire mathematical literacy and grasp the importance of mathematics. Maryville's model of teacher as reflective practitioner requires solid grounding in liberal arts, mathematics, and science, and it espouses a constructivist framework. Students are immersed in experiences as mathematical learners and encouraged to share their mathematical knowledge and conceptions. The program assumes students will learn content and pedagogy simultaneously. Students are encouraged to question instructors' objectives, methods, and curricular and pedagogical decisions. Maryville's instruction seeks to: (1) promote autonomy and commitment; (2) develop reflective processes; (3) construct case histories; (4) identify and negotiate solution paths with students; (5) retrace solutions paths; and (6) adhere to the intent of the materials. Empowered, assertive students leave the program understanding the value of mathematics. They are able to plan for technology to be part of their mathematics program, and they understand they must construct nontextbook experiences for their students. (SM)

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Mathematical Literacy to Empower Teacher Education Students  
in the 21st Century: How Can This Become Reality?

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Presented at the AILACTE Fifth National Forum, June 6, 1992

## Mathematical Literacy to Empower Teacher Education Students in the 21st Century: How Can This Become Reality?

### A Time for Change

In the latest of repeated blasts, diatribes, and calls for reform in mathematics education, the Carnegie Commission of Science, Technology and Government (1991) states that "every school day, students in these (elementary) grades come to school naturally curious about the world and go home having learned to hate science and mathematics a little more.

The call for mathematics reform (this time around) has been more saliently presented than the "new Math" of the 60's. It is even more critical that mathematics education be reconceptualized and rethought than at any other time in our history.

Why is it critical to reconceptualize mathematics teaching and learning on such a large scale at this time? Some of the major changes that affect this call for reform are outlined in Reshaping School Mathematics (1990) and are as follows:

Changes in the need for mathematics.  
Changes in the mathematics and how it is used. (p. 1-2)

Decisions about one's life in society hinge more and more upon automated, technological procedures that are driven by value-laden decisions about the models by which individuals live, work, receive health care, and acquire goods and services. Mathematical models are used to organize and manage almost every facet of an individual's existence.

Changes in the role of technology.  
Changes in American society. (p. 2-3)

The use of computers and calculators has spread to almost every workplace and, to a certain extent, every job. The technology has changed each individual's ability to do mathematics and analyze data at a rapid pace.

At this point in time, students who have not had adequate background in mathematics or have developed significant anxiety about mathematics will simply be denied access to many occupations. Mathematics based courses such as statistics, accounting, economics, and finite mathematics are the so called "gate keepers" to students' success and access to degrees and occupations. The spillover of mathematics into the management of our society must, then include scientific and mathematical literacy for all.

Changes in understanding of how students learn. (p. 3)

Based in large part on the theoretical underpinnings provided by Piaget, other researchers and classroom teachers (e.g. Kamii, 1985) have come to a greater understanding of how children (and for that matter adults) must actively construct their own knowledge. This knowledge is not just factual, but includes active organization and analysis of experiences, new data, and previously learned mathematics, both individually and in a social setting with others. Children come to any school experience with a great amount of past experience and mathematical knowledge that can and should be brought to bear every day in the classroom.

Changes in international competitiveness. (p. 3)

Students in the U.S. do not seem to be performing as well in mathematics as some of their counterparts in other countries. "In particular, most other industrial countries have considerably different expectations about topics taught and level of performance than is common in American schools." (p. 3)

As the Education Division at Maryville University (and as a country), we must be asking about the initial and continuing preparation of our teaching force in a discipline that has been among the most misunderstood and feared by many teachers.

#### Teacher Education and Mathematics Education at Maryville

What can we assume about many of our entering preservice elementary school teacher education candidates? In what ways will their conceptions and ideas about mathematics influence their curricular and instructional decisions in this area?

In the Maryville University teacher education program, we have made the assumption that our program will train competent beginners who believe that ALL children can learn. In the case of mathematics, what often happens is that they must first be convinced that they can learn it themselves. In the case of the minority of these students who are more mathematically successful and/or sophisticated, we still assume that we must challenge the assumptions about the very nature of what mathematics is as well as how it is taught.

In terms of past mathematics preparation, the students vary. Most, however, do not have much preparation beyond high school geometry. While the age of our teacher education students varies from 19 to mid-50's, there is alarmingly little difference in the way in which they perceive and recall their own mathematical preparation.

When they come to us as sophomores, in their first semester of teacher preparation we assume:

- a. More than half of the class does not like mathematics and has not had positive experiences in their own mathematics learning. (AAUW, 1991)
- b. Most perceive that one must have a "mathematical" mind. Few have reflected upon themselves as a learner of mathematics. (AAUW, 1991; Kenschaft, 1991)
- c. Few have given any thought to the way in which they will teach mathematics, other than the computation and drill they themselves have experienced. (Thompson, 1984; Kennedy, 1991 a,b)
- d. Though they own a calculator, they do not know how to use many of the function keys on the calculator. Many have no experience with computers except for word processing.
- e. Mathematics is perceived by them to be: a study of numbers, a smattering of geometry, computation, and difficult, boring, or anxiety producing. (Thompson, 1984)
- h. The primary pedagogical model to which the students had been exposed is a direct, skill-oriented "show, tell, and practice" model. Paper and pencil are the primary tools to learn. (Hollingsworth, 1989, Confrey, 1987 cited in Confrey, 1990)
- i. Students exposed to the above mentioned model will teach as they were taught, until they construct for themselves other models of meaningful teaching. (Kennedy, 1991 a,b)
- j. Much of what we are trying to teach our students about the learning and teaching of language can be connected to the learning and teaching of mathematics and other subject areas. (NCTM, 1989,1991).
- k. Mathematics learning can be integrated with the learning of other subjects. (NCTM, 1989, 1991).
- l. It is imperative that students "get a grip" on mathematical literacy and buy into the importance of mathematics learning for each and every one of their students. (NRC, 1989, 1991)
- m. The climate created by us in their mathematics and education courses will be critical if we are to change their belief and influence their teaching practice. (Confrey, 1990; Maher & Alston, 1991, MAA, 1991)

### A Constructivist Programmatic Model

The division's model of teacher as reflective practitioner (Maryville University, 1991) requires a solid grounding in liberal arts for our students. It espouses a constructivist framework for the beginning and ongoing development of this practitioner. Our division decided long ago that we would make sure that the liberal arts grounding extended beyond humanities, social science and the arts. We realized that we would need to ensure that the mathematics and science preparation of our students would increase the student's breadth and depth of the knowledge that they would need to teach mathematics.

As a result, our students are required to take 12 hours of science (3 lab courses) and 8 hours of mathematics. The courses that they take were jointly designed by our mathematics and science division faculty and the mathematics and science educator within the division. Special attention was given to topics, sequence, and lab experiences that would address areas typically taught by elementary school teachers.

We are also informed by the leadership in the area of mathematics reform provided by the MSEB (1990,1991), NCTM(1989,1991), National Research Council (1989), and MAA(1991 a,b). Our own mathematics department has not chosen to study these recommendations, but, in a small liberal arts college, they have allowed me to cross ranks in a rather non-traditional way. Several years ago after my involvement in a cooperative venture with the head of the mathematics department, I was invited to teach the first of the mathematics courses for elementary majors, hire the faculty member for the second course, and follow up with the teaching of the mathematics methods class myself. By keeping pedagogical and philosophical consistency across the students' mathematics program and teacher education program, students experience a more unified, integrated preparation.

Simultaneously, the education faculty is involved in the integrated team teaching and planning several blocks of professional education courses. As a result, students have the opportunity to see me teaching in two different areas and are encouraged by all of us to compare and contrast what we teach, how we teach it, how we make instructional decisions and how I personally reflect upon my teaching of mathematics and education classes. There are, then, several aspects of the way that these courses are taught that are critical to the empowerment of these students. First and foremost among these is that each of us model the teaching and reflection for our students that we are asking them to do themselves.

### Fostering a Constructivist Approach to Teaching Mathematics

To begin the development of mathematics teachers, then, we



start by immersing students in experiences as mathematical learners, knowing that the constructivist perspective that we espouse "has dramatic implications for mathematics instruction (Confrey, 1990, p.111)." Confrey (1990) suggests that "we must promote in our students the development of more powerful and effective constructions " and that " the most fundamental quality of a powerful construction is that students must believe it (p.111)." "The view that mathematical learning is a process of active construction locates the source of meaning in students; purposeful, socially and culturally situated mathematical activity." (Cobb, Yackel & Wood, 1992) This would imply that specific mathematical representations are present within the individual and not within an external, arbitrary symbol system. Yet, simultaneously, the view acknowledges "that knowing is a socially and culturally situated constructive process" (Cobb, Yackel & Wood, 1992, p.8). Interpretation of the symbols that describe mathematics, as presented by either teacher or student, then, are inadequate to represent students' understanding. This is true of symbols used on papers or elaborate technological representations as well.

Our commitment to this perspective then, implies that we will be helping our students first make powerful mathematical constructions themselves and then, that

An instructor should promote and encourage the development for each individual within his/her class of a repertoire for powerful mathematical constructions for posing, constructing, exploring, solving and justifying mathematical problems and concepts and should seek to develop in students the capacity to reflect on and evaluate the quality of their constructions. (Confrey, 1990 p. 112)

Students must be encouraged to share their informal mathematical knowledge and conceptions of mathematics. They must also share the strategies, insights, intuitions, and feelings as they explore things mathematical. Rather than deny past experience, they must compare and contrast what goes on presently in their mind. We explore the "myths" of mathematics. The first two weeks are critical, in that they are spent redefining the students conceptions of mathematics, their own mathematical learning and the ways in which they communicate about mathematics. Students are encouraged to share strategies and solutions to open-ended, real life types of problems and exercises that help them perceive how their past knowledge of mathematics influences the ways in which they now think about and organize mathematical understanding.

Often in mathematics, it is assumed that through the teacher's use and understanding of symbols and the required replication of the child's use of those symbols, mathematical

understanding by the child is achieved. We are trying to produce teachers who understand and act upon the fact that this is simply not the case.

### Specifics of the Mathematics Courses and Instruction

So, how do we do this? The following outlines some of what we do. We use a sequence of delivery that extends (for most) over a 3 year period of time. In Year 1, students take a two semester sequence of courses that work them through the structure of the number systems, the nature of thinking about mathematics, geometry and spatial development and the organization and management of data. Practica during each of these semesters also give students the opportunity to reflect upon the mathematics being taught in schools. In year 2, the mathematics methods course is taught integrated with children's literature, reading methods, language arts methods and a practicum. In year 3, a full, 16 week student teaching experience and accompanying seminar provide opportunities to develop and refine teaching.

Specific activities to use are often adapted from the research work of Carpenter and Fennema (1988), Wood, Cobb, and Yackel, (1991), Kamii (1985). In addition, practitioners who are experimenting with this and reporting on their successes annually at NCTM conferences have been a source of ideas. The work, films, and writings of Marilyn Burns (e.g. 1992) have provided many situations that can be used and insights from hundreds of teachers who are trying to become more constructivist in their teaching.

Students are encouraged to become mathematical learners again. The speaking about, writing about, and interacting with concrete materials about mathematics are encouraged, valued and debriefed as they are occurring. Frustrations, confusions, misgivings, reconceptions are all valued and encouraged during class time and in independent work and writing. Many of the tasks that are used can be used with children as well. There is no need to assume that certain tasks should be left only to children or to adults. For example, the early study of patterning with numbers, geometric shapes and a search for other instances in which patterning is important in every day life uses many of the same activities that are used with sixth and seventh graders. The level of mathematical understanding and insight provide rich alternatives and multiple strategies for people to share. Purposefully, MOST of the tasks used are time consuming (30 minutes-1 hour each) and involve multiple strategies and multiple solutions.

Dissonance in both the teaching and curriculum of the courses is deliberate and planned. Tasks are chosen so that what might appear immediately obvious becomes unclear or unobvious as the complexities of the task are unfolded. Over and over,



(throughout all 3 years of the program), students are asked to consider that "confusion is good." For students whose perceptions of mathematics are most often based upon "getting the right answer", this redefining and reexamining the purposes of confusion in light of developmental processes is perhaps one of the most powerful things that is done. Students are reminded that they are often willing to come to a literature class being ready to discuss things that confuse or puzzle them, but that they do not often allow themselves the same luxury in their mathematical development.

Deliberate attempts are made to help students value the fact that the mathematical processes that they use are as important as "the right answers." Students are challenged from the first day with situations for which there are multiple solutions or value laden, multifaceted choices that must be made before a solution can be found. For example, a task presented to them at the very beginning of their instruction is to figure out how much it would cost to make and transport pizza for lunch for every person in the school in which they are placed for a practicum. Solutions must be presented to the entire group with a narration of how the group came to the solution, what estimates they made, and what decisions had to be made as well as a rationale for those decisions.

Reflection upon the depth of information provided by attending to process rather than simply final solutions is mind boggling to these students as they begin to understand how this affects teaching and learning. The gradual progression that they go through in valuing and looking for knowledge of process in their own instruction and that of their cooperating teachers has been very powerful.

In an effort to pull students out of their conceptions of common understanding about computation, exercises involving mental math and estimation are used on an almost daily basis. After helping students explore and understand that much of their knowledge about computation and number sense is based upon intuition, mental mathematics, and estimation rather than paper and pencil calculations, what remains is providing opportunities for students confidence and insights into their ability as mental computers. Next, of course is the creation of dissonance for them as they conceive of current curriculum and instruction in the elementary schools.

#### Where Does Pedagogy Fit?

Perhaps one of the most unique (and most intriguing aspects) of our program is the assumption that the student will learn content and pedagogy simultaneously is directly discussed and reflected upon by all of us during each class session. Students are encouraged to question the instructors objectives, method of instruction, curricular and pedagogical

decisions as well as those of the teachers who they are observing in the schools.

I often stop class early to reflect upon what has happened. From my own experience and the feedback of my students, this is most valuable. Emphasis is upon what I am seeing and hearing from the class, how I began class not knowing exactly what would happen or how it will unfold, how my instruction occurred based upon their responses, and any depth vs. breadth frustrations I might be feeling. My goals and objectives are also discussed and critiqued by myself and my students.

The feelings and frustrations of individuals and groups are encouraged to be openly shared. This includes the "strong" student frustration with the pedagogy being used at first. Another focus of discussion is the ways in which all students change in conceptual knowledge and how this occurred. A real "intestinal fortitude" is necessary for about 6 weeks on the part of the instructor. Students express the desire to go back to old, "safe" ways of doing things. It usually takes that long for students to really believe that what they are doing and experiencing will be valued in a different way. We discuss openly and frequently what kind of atmosphere must be present for meaningful learning to occur. Personally, breakthroughs begin to be measured as students become willing to share incomplete or misguided procedures voluntarily.

Students write about their mathematical understanding. The journaling that occurs throughout each of the semesters provides an individual forum for the student and a record of growth. All assessments and tests include written narrative about process.

One has to find pedagogical ways to rechallenge student learning to help them see that much of their skill development and learning gets in the way of or is not grounded in solid conceptual development. A good example occurs as students are asked to reconsider place value. I give them examples from the Mayan and Aztec number system, only one of which uses place value. With no further instruction, I ask them to "crack the code" and then let them spend the two classes necessary to come to a common understanding. In addition, this exercise is useful to help them see how one's initial understanding may become clear and then regress to a more unclear state. This helps to challenge the notion of "mastery" as it relates to continually developing concepts.

Students come to grips with the fact that their own mathematical understanding is not linear. The typical students image of mathematics as a discipline is that it is very orderly, very sequential and very hierarchical. For example, based upon their past practice, they perceive that

one must "master" addition before moving to subtraction. These assumptions are challenged through the examination of their own computational facility and what we know about the incidental mathematical learning of young children that occurs before any formal education. They come to realize that their own understanding is much more contextual and experience based than they had realized in the past.

Much of their knowledge of mathematical functions and operations ceased after they learned whole number computation. Instead, they tend to define any further understanding on their paper and pencil computational facility. By asking them to demonstrate and explain various models for each of the operations with whole numbers, they find new models that Carpenter and Romberg have pointed out are usually not present for elementary teachers. Then, they are challenged to justify how the models of operations can or cannot be extended for other number systems such as integers and rational numbers. When I innocently ask "Why do you get a smaller number than each of the factors when you multiply  $0.9 \times 0.1$ ; I thought you always got a bigger product when you multiply, they sigh, groan and know that we are in for several days of heavy duty work. Few have conceptual underpinnings or explanations for anything that they do with the decimal or fractional number system. We spend much time exploring these ideas with manipulatives and calculators.

By rebuilding number systems and using mental math, students realize that their computational facility does not imply that they understand what they are doing and why they are doing it. They also realize that much of their life experience in which they use mathematics is not informed by the traditional school experience.

Careful use of manipulatives guide almost every experience. There is great emphasis put not only on what is used but the way that they are used, noting that direct instruction using manipulatives can also occur and is not appropriate for what we are trying to do. We use sticks, buttons, attribute blocks, place value blocks, Cuisenaire rods, discs, tiles, geoboards, pentominoes, blocks, geometric solids, and various and assorted "stuff". Students are taught where such materials can be purchased as well.

The development of knowledge about the crippling effects of mathematics anxiety and ways to combat it emerges every semester. This is not at all surprising, particularly because of the number of women enrolled in these courses.

At each step along the way, they are encouraged to ask "Why is this important? How is my own learning of this going to affect my teaching? How will I keep make sure that I allow opportunities for the mathematical understanding of each student to be used and valued?" They leave their first two

semesters having "experienced" mathematics in the making for each and every one of them. They then turn to the methods course and practicum where they do get a chance to try this for themselves.

Technology does play an important role in developing understanding. Students are taught to use calculators and then allow to use them at any time. Some computer software is used, but the criteria for its use are carefully considered. The software must also allow for the construction of new knowledge. We find that the software from Sunburst and Wings for Learning is most valuable.

### The Constructivist Model used for these classes

In summary, then, the instruction provided seeks to be a model with the six components so saliently summarized by Confrey (1990):

1. Promotion of autonomy and commitment in the students;
2. Development of students' reflective processes;
3. Construction of case histories;
4. Identification and negotiation of tentative solution paths with the student;
5. Retracing of those solution paths; and
6. Adherence to the intent of the materials. (p. 115)

Establishing the right atmosphere and tone in these classes is essential in order to foster student autonomy. One must truly value student thought processes and feelings and model that much more than "the right answer" is of value in these classes. This fostering of autonomy also means that the students must learn to rely on their own thought processes and those of their fellow students. They find this process empowering and the growth in their confidence is truly remarkable.

### The Methods Class

Students come to methods classes that are totally integrated with the reading/language arts/children's literature courses. In addition, all 3 of the instructors (including myself) supervise a practicum that occurs for 3 mornings/week, during which they will design, implement, evaluate, and reflect upon their own teaching. Cooperating teachers are hand-picked for these experiences and given training in

supervision. Typically, our students are willing to experiment with literature sets, but express the reservation that they perceive that the cooperating teacher is not as willing to allow them to experiment with mathematics. This is usually a perception that can be cleared up with a 3 way conference with the teacher.

In the methods class, many assumptions and experiences are revisited. Developmentally appropriate materials and experiences for different grades levels are discussed and sample lessons and videotapes help students make sense out of what will work and help in constructing mathematical understanding in children. Each student is informed by exploring journals including their own minisubscription to The Arithmetic Teacher and critical reading of the Curriculum and Evaluation Standards for School Mathematics (1989) and the Professional Standards for Teaching Mathematics (1991). Resources from the professional organizations and commercial publishers are explored and discussed.

Technology plays a very important place in this class. In particular, it would not be the same without the videotapes used (particularly those of Kamii (available from NCTM) and Marilyn Burns (available from The Math Solution)).

Students most often express willingness and enthusiasm for the teaching of mathematics. The convincing them of its importance is no longer necessary. At the same time, they express reservations about how they will put into practice some of what they have experienced themselves. They must also refresh their own thinking about experiences that were crystal clear a year before. This is extremely valuable as they experience themselves the need to recycle old concepts with new experiences. Those assigned to teach mathematics in an upper grade practicum experience are typically a bit more nervous than those assigned to a primary setting.

Much of the methods class becomes critical questioning to help students construct instruction and curriculum of their own that is consistent with the assumptions about mathematics learning that we have explored together and their own personal goals regarding their teaching style. It is critical to this process that we attend to what we know about teacher development, as has been explored extensively by Kennedy (1991) and Berliner (see Berliner, Stein, Sabers, Clarridge, Cushing & Pinnegar, 1988; Fuller, 1969). We realize that we are working with novices and that the ways in which they approach teaching are vastly different from expert teachers or even more advanced beginners. In essence, their construction of mathematical knowledge and their construction of teaching are proceeding along simultaneously. The sheer complexity of this consideration has been mind-boggling for us as well as our students.



## Lesson Planning

In particular, these beginners' lesson planning needs careful consideration for us with attention to the following areas.

These novices are very concerned with their own performance, first and foremost (Fuller, 1969). One of the questions I hear most often is "What am I going to do?" They have to be reminded to consider the children's understanding and exploration in their planning.

While they are willing to do this with help, our students can regress to very direct model lesson planning if they are not comfortable with the curriculum or sense that direct instruction is typically used by their cooperating teacher. Encouragement and questioning from us (as well as review of their lesson plans) helps us to encourage them to be more constructivist in how they operate as teachers.

A tension between their willingness to attempt to be constructivist and their fears about classroom management and control almost always exists. This is an area about which the Division faculty need to do more research. Constructivist teaching requires careful questioning, skills in leading discussion, and the ability to move in the direction of student thinking as the lesson is occurring. It can also be facilitated by previous experience with the same tasks in order to anticipate at least some of the questioning and thinking that might occur during the class. Assisting beginners with this type of instruction when they are not yet experienced in the types of responses they might get or the management necessary for such lessons can present challenges if support and extensive supervision are not present.

One way that we have learned that we can help this and most directly affect their practice is in our own modeling of lessons and in providing them with some written lesson plans that are constructivist lessons and then encouraging them to "steal them" and use them. This has been successful in language arts and reading as well as mathematics.

This "construction in process" can be frightening for our students and can result in marvelous opportunities to reflect upon the lesson afterwards, but left to reflect on their own, our students may be disheartened by the results of their lessons. This is important to note in addition because these students have had extensive field experiences. In fact, this practicum occurs in their fourth semester, having already had teaching experiences in semesters two and three. The role of the supervisor is absolutely critical. Asking students critical questions and helping them process their planning and implementation cause the student to develop his/her teaching in a way that cannot be done without such reflection



Constructivist teaching does not occur on a one day basis. Therefore, the evaluation and reflections on their teaching must be assisted so that they look at a "lesson" and its effectiveness in a long-term way.

### Tensions in the process of learning to teach mathematics

Our students run into trouble "plugging in lessons" in areas that the teachers in the classroom have been teaching more directly. One common example occurs when the classroom teacher has developed the rote, standard algorithm for long division and the students in our program know that it should be taught more constructivistly, allowing for and encouraging variations in the procedures for students. Students are reluctant to confront cooperating teacher practice; those of us who supervise move gradually to attempt to influence practice in this arena as well. We have had some notable successes, but we have also had to move to new practicum sites when support for our students has not been forthcoming, even after 3 years of work with the cooperating teachers.

In addition to the emphasis on constructivism in mathematics, this semester asks students to look at alternatives to traditional assessment and try to implement some of them. Our students seem to do a better job of this each year. The school climate and interest for this assignment grows each year as well.

### Reflection upon Instruction

We also ask them to reflect upon and analyze their instruction to all children. We do push them to articulate how each child learns in the classroom and ask them to reflect upon their attitudes and treatment of children who are female, Asian-American, African-American, poor and diagnosed as having learning disabilities. Our students articulate a willingness to teach all children for the most part; they continue to need to be pushed to define and assess success for each child. They continue to look at class reactions to a lesson rather than focusing on the feedback from individual children. While we continue to work on this, we also realize that we are asking for a lot from beginners.

Videotape feedback and analysis provides us with individual and group opportunities to process through instructional decisions, teaching decisions, and the professional development of each of our individual students. Examples of analysis (also videotaped) include salient reflection upon goals and an understanding of whether or not each of our students has enhanced the mathematical understanding of the students.

Continued questioning in class by the students and the instructors gets them to analyze each discipline and begin to help these students through the transition to more constructivist teaching. Their questioning and analysis (which takes place in a non-graded practicum to remove one potential threat to creativity and experimentation) improve throughout the semester. Journaling provides reflection upon lessons taught. Written reflections upon reading for the course help to delineate questions that students are having about both content and instruction. Our students are articulate about their own teaching and mathematics instruction in the school. They leave the class believing that they can make a difference. They also leave the class needing continued support and feeling the documented pressure of reconciling what they observe in practice and what they perceive about best practice through us.

Student teaching supervision at Maryville University is all done by full-time faculty. This means that the supervision is consistent with the program and that which has been done at earlier stages. These elementary students are not afraid to teach mathematics. They repeatedly return to borrow those materials and manipulatives to which they have had access throughout the program. Teachers rate them highly in their preparation in this subject area. Again, we also match student teacher and cooperating teacher and continue to try to ensure that the cooperating teachers understand and help us operationalize our model.

#### Results for our students

Our results from this have been gratifying. In reality, many of the results of this work have been shared in the commentary up to this point. In addition, we have evidence from both standardized tests, classroom tests, and other assessments that our students' mathematical knowledge is greatly enhanced throughout their experiences with us. In addition, our students are confident about their ability to teach mathematics. At the student teaching stage, almost all of our students choose to teach mathematics very early in their placements and video these lessons as well. In student teaching seminars, many examples that they use are related to mathematics. They also have been able to influence the practice of their cooperating teachers. Even in anonymous evaluations and one and three year on-site visits and returned questionnaires, they highlight their preparation in mathematics. Administrators corroborate their excellent preparation.

Our preservice teachers' development is just beginning. Explicit in our model is the notion that ongoing professional development will occur for the teacher as professional and in the areas of curriculum and instruction. We hope that this solid foundation in content, curriculum, and instruction will

continue to carry through in the students' careers. We also know, however, that without ongoing support, students will regress to more traditional instruction. Our own institutional follow-up (on site) seeks to help keep the programmatic model we have developed with them at the forefront of their thinking.

Our first steps toward this teacher development occur within the program. Mathematics education in the elementary and secondary schools in which our students will teach is undergoing a fundamental change. Our program should enable our students to be able to continue to reflect and move forward in their development in these areas. The following can be said about the transitions called for in the entire mathematics community that summarize where we perceive both our students and the cooperating teachers as they continue their mathematical and professional development. The points made are highlighted in Everybody Counts: A Report to the Nation on the Future of Mathematics Education (NRC, 1989):

1. The focus of school mathematics is shifting from a dualistic mission--minimal mathematics for the majority, advanced mathematics for a few--to a singular focus on a significant common core of mathematics for all students.

2. The teaching of mathematics is shifting from an authoritarian model based on "transmission of knowledge: to a student-centered practice featuring "stimulation of learning."

3. Public attitudes about mathematics are shifting from indifference and hostility to recognition of the important role that mathematics plays in today's society.

4. The teaching of mathematics is shifting from preoccupation with inculcating routine skills to developing broad-based mathematical power.

5. The teaching of mathematics is shifting from emphasis on tools for future courses to greater emphasis on topics that are relevant to students' present and future needs.

6. The teaching of mathematics is shifting from primary emphasis on paper-and-pencil calculations to full use of calculators and computers.

7. The public perception of mathematics is shifting from that of a fixed body of arbitrary rules to a vigorous active science of patterns.

Some of our analysis of our results have seemed to point to success. Again, however, we wish to emphasize some of the

important processes that we believe contribute to the successful development of our students as mathematicians and mathematics teachers.

We are willing to move slowly and look for incremental success in our students.

We persist in modeling constructivist teaching and learning and being very inductive in our teaching of both. We are very well aware of the time commitment involved in this and know that we as faculty must continue to develop our ability to carefully and constantly question ourselves and our students about their learning and teaching. We are committed to an emphasis upon the social aspects of teaching and learning mathematics. Through questioning and discussions with others, everyone's understanding is enhanced.

We look explicitly and extensively at traditional and alternate forms of assessment. Our own teacher education program is requiring students to develop portfolios. Students use reading, writing, listening, and video assessments as they teach. In addition, the development of a case study in reading, language arts, children's literature and mathematics helps our students analyze the overall progress of an individual child and prescribe the instructional steps that they would take next with that child.

We are committed to developing alliances with schools and teachers who are also committed to continuing reform of practice and ownership in their own professional development. Part and parcel of this is the continuance of our encouraging and valuing the criticism of the practice that our students are seeing in the schools. Mutual trust and respect are developed through careful selection and nurturing of individual teachers, principals, and schools. In addition, we strive to maintain rigor, reference to research, and objective data that corroborate both accolades and criticism of current practice. We strive to ensure that the context of schooling in the community and society inform and help to dictate best practice.

Our students hold us accountable for modeling strategies, use of materials, and taking class time to discuss why something was taught the way that it was, and what pedagogical and curricular choices were made. This professional responsibility on our part is paramount. We believe that this is probably our highest professional responsibility.

We acknowledge that the breadth vs. depth tension that they feel is one also felt by me and my colleagues as I teach. This is felt in assessment and instruction. Mathematics education research informs my practice and speaks to this as

well.

Research in mathematics education guides us all along the way. Students often join NCTM on their own. They continue to look to the professional publications of the organization to inform their practice. This includes the fact that students try to make mathematics more meaningful and tied to real life experience. There is also evidence that they are able to translate the assessment practices used with them into practices used with children. They also continue to see that curricular decisions informed by research and constructivist thinking look to organize some tasks as stage setting, developmental, and long-term in their usefulness for further mathematical understanding.

We strive to make students believe in their own mathematical power and ability by helping them monitor their own self-confidence and reasoning. We seek to infuse in them the desire and willingness to appreciate the importance of mathematical success for all children. We help them develop as teachers by dealing honestly with their own development when they ask questions like, "What if the other teachers don't do it this way? What will my kids do when they go to teacher X?" We also constantly seek out models for field experiences that allow students to experiment with some of what they have seen us do in class. Our graduate students have been immensely helpful in this area. They have opened As a result we have also been able to invest in field sites and teachers over the long haul, enhancing our program and our cooperating teachers professional development.

### Conclusions and Questions

We believe that impacting practice occurs through the long-term attention to content and pedagogy simultaneously. It also seems that we are able to remain true to a constructivist perspective only by teaching in a constructivist manner ourselves. The ongoing, long-term linkages between our students mathematical learning and their teaching provide a powerful foundation for their ability to teach mathematics in a meaningful way.

Our students value mathematics and see its importance as they leave our program. One of our major concerns is that we cannot ensure support for this teaching model in the critical first years of teaching. These students do, however, remain in touch with us to ask questions, look for materials, and borrow the manipulatives that have been used with them.

Students leave us planning for technology to be part of their mathematics program. They also assume and accept that they will need to construct non-textbook experiences for the children who they teach.

Our students leave more assertive about the mathematical needs and less fearful about the teaching of mathematics. This confidence is carried to the children who they teach, all of whom will need to experience success and be confident in their own mathematical abilities.

Continued research is needed into the ways in which we help our students change their conceptions of teaching and learning. In particular, we need to understand how our students experience constructivism and, at the same time, learn to be constructivist in their teaching. The sustained programmatic emphasis on this model at Maryville seems to suggest that our methods and the reflection of ourselves and our students on those methods fosters deeper understanding of content and teaching.



## REFERENCES

- AAUW (1991). Shortchanging girls, shortchanging America. Washington, DC: Author.
- Ball, D.L. & Wilson, S.M. (1990). Knowing the subject and learning to teach it: examining assumptions about becoming a mathematics teacher. East Lansing, MI: National Center for Research on Teacher Education.
- Berliner, D.C., Stein, P., Sabers, D., Clarridge, P.B., Cushing, K., & Pinnegar, S. (1988). Implications of research on pedagogical expertise and experience for mathematics teaching. In Grouws, D.A. & Cooney, T.J. (Eds.), Effective Mathematics Teaching. Reston, VA: National Council of Teachers of Mathematics.
- Borko, H., Eisenhart, M., Brown, C.A., Underhill, R.G., Jones, D., & Agard, P.C. (1992). Learning to teach hard mathematics: Do novice teachers and their instructors give up too easily? Journal for Research in Mathematics Education, 23(3), 194-222.
- Burns, M. (1992). About teaching mathematics. Sausalito, CA: Math Solutions Publications.
- Carpenter, T. P. , & Fennema, E. (1988). Research and cognitively guided instruction. In E. Fennema, T.P. Carpenter and S. J. Lamon (Eds.). Integrating research on teaching and learning mathematics (pp. 2-19). Madison, WI: Wisconsin Center for Educational Research, University of Wisconsin-Madison.
- Confrey, J. (1990). What constructivism implies for teaching. In Davis, R.B.; Maher, C. A. & Noddings, N. (Eds.) Constructivist views on the teaching and learning of mathematics. Reston, VA: NCTM.
- Cobb, P., Yackel, E. & Wood, T. (1992). A constructivist alternative to the representational view of mind in mathematics education. Journal for Research in Mathematics Education, 23(1), 2-33.
- Kamii, C. K. (1985). Young children reinvent arithmetic. New York: Teachers College Press.
- Kennedy, M. M. (1991) Policy issues in teacher education. Phi Delta Kappan, 72 (9), 659-665.
- Kennedy, M. M. (November, 1991). Some surprising findings on how teachers learn to teach. Educational Leadership, 49(3), November, 1991.

Kenschaft, P.C. , Ed. (1991). Winning women into mathematics. Washington, DC: Mathematical Association of America.

Maher, C.A. & Alston, A. (1990). Teacher development in mathematics in a constructivist framework. In Davis, R. B.; Maher, C.A. & Noddings, N. (Eds.) Constructivist views on the teaching and learning of mathematics. Reston, VA: NCTM.

Maryville University (1991). Becoming a reflective practitioner: teacher education at Maryville University. St. Louis, MO: Author.

Mathematical Sciences Education Board. (1990). Reshaping school mathematics: A philosophy and framework for curriculum. Washington, DC: National Academy Press.

Mathematical Sciences Education Board. (1991). Counting on you: Actions supporting mathematics teaching standards. Washington, DC: National Academy Press.

National Council of Teachers of Mathematics. (1989). Curriculum and evaluation standards for school mathematics. Reston, VA: Author.

National Council of Teachers of Mathematics (1991). Professional standards for teaching mathematics. Reston, VA: Author.

National Research Council (1989). Everybody counts: a report to the nation on the future of mathematics education. Washington, DC: National Academy Press.

National Research Council (1991). Moving beyond myths: revitalizing undergraduate mathematics. Washington, DC: National Academy Press.